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Multi-functional formalized quantum circuits

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Abstract - To become practically and functionally viable quantum computers they should be able to use a technology for formalization of a set of operators that is of vital importance to be able to mimic the behavior of the quantum parallelism. In this study is offered a methodology to perform mapping of the quantum algorithms for designing of effective formalized quantum hardware schemes. The model is designed with consideration for implementation of superpositions and entangled input states. The proposed models provide guidelines to conduct accurate researches based on formalizations of quantum hardware schemes for practical quantum applications.

Key words: Quantum computing, circuit, operators, gates

1. INTRODUCTION

Unlike the fixed models, the formalized models circuits maintain an unlimited number of operators. The functionality of the formalized circuit can be altered simply by changing the angles values of the rotational gates in the circuit. In this work we present a new technique for a model of quantum circuit with a result two basic formalized circuit schemes. The schemes of the circuits can be used for simulation of a certain operator by adjusting the values of the angles in the circuit. This provides a model with fixed-circuit, whose angles are determined by the elements of a given matrix, which may be non-unitary in an effective way.

The classical logical devices in a broad sense are classified as fixed and formalized devices. As understood from their name, the circuits in a fixed logic maintain only one function that is determined at the time of their creation. This cannot be changed at a later stage. On the other hand, the formalized devices as PLD (programmable logic device) and FPGAS (field-programmable gate array - programmable logical matrix) are able to support an unlimited number of functional capabilities, as they can be reconfigured outside the production environment. With such a characteristic the designers and programmers can manage and simulate their test models and algorithms.

The quantum computing is turning into a huge new inter-disciplinary area by providing a variety of approaches and protocols to various subfields such as: communication, encoding, global binary optimization (see adiabatic quantum computation [2]), linear algebra etc. [3- 5]; however, the formalized quantum circuits and the design of the chip like those in the classical computers remain an open issue.

In the model of the circuit of the quantum computing the operators of unitary matrices represent algorithms or a certain part of the computations [6]. From here one of the fundamental issues is to have a multifunctional quantum circuit or quantum chip that to be able to realize different types of algorithms in a quick and effective way. The possibility for designing of multi-functional quantum logical matrices as a multifunctional quantum computer is discussed in reference [7]. It is know that the logical matrix can be programmed for calculation of an expected value of a given operator [8]. For the quantum gate realization is offered a cell-structured model of a quantum circuit, based on the activation and deactivation of the gates on different qubits: The combination of similar cells can be used for realization of a given sequence of quantum gates[9]. Moreover the various schemes of the main formalized multifunctional quantum circuits are shown for two [10,11] and three qubits [12- 14], found by applying different schemes for decomposition to a given unitary operator. On the basis of the main two-qubit model of a circuit experimentally is realized a two-qubit quantum processor [15]. However, the realization of a main quantum processor and a full scale quantum computer still represents an obstacle that requires new theoretical and experimental improvements.

It is known that the realization of quantum logic operations can be simplified by the use of the higher dimensional Hilbert spaces [16, 17]. In this report, using ancilla qubits, we describe a new approach for designing a circuit that generates two formalized quantum circuit models. They can further be improved in order to design basic scale quantum chips and formalized quantum logical matrices. The circuits also support simulations of non-unitary matrices. We also present analysis of the complexity for the circuits: with a view to the quantum complexity, they have the same complexity as non-formalized models, which are generated by the use of matrix decompositions in the numerical linear algebra as QR decomposition [18], the quantum Shannon decomposition, the cosine-sinus decomposition and some others [19,20]. With regard to the classical complexity, since the angles for our formalized circuits can be defined simply as individual matrix elements, the classical complexity is much simpler than the methods of decomposition.

This report is organized as follows: After the introduction of the main idea for simulation are presented the details of the two circuit model that implement it. Then the complexity of the circuits is analyzed in terms of the classical and quantum complexity. Finally, we discuss the circuit models and possible future directions. In the Annex are presented more computing details related to the matrices.

2. THE MAIN IDEA FOR SIMULATION

For a given real unitary $U^{N\times N}$ with $N = 2^n$ n is the number of the qubits, the link between the input data $|\psi\rangle = \alpha_1|0\ldots0\rangle + \ldots + \alpha_N|1\ldots1\rangle$ and the output results $|\phi\rangle$ is defined as $|\phi\rangle = U|\psi\rangle$ generating N states:

$$U|\psi\rangle = \begin{pmatrix} U_{11} & \cdots & U_{1N} \\ \vdots & \ddots & \vdots \\ U_{N1} & \ldots & U_{NN} \end{pmatrix}\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix} \tag{1}$$

Any system of a higher value (the ancilla qubits are added to the original system) can also be used for generation of this result on N selected states with a certain normalization. Our goal is to create a matrix V (shown in equation 2), which represents the system with the ancilla qubits. Then we modify the initial input data $|0\rangle|\psi\rangle$ to this extended system V (the initial state of the ancilla qubit is taken as $|0\rangle$) through the use of quantum operations, so that the application of V to these modified input data $|\psi\rangle$ to include the result, given in equation (1) with a normalization constant k: (2), where each V_i possesses special rows of U as leading rows. The addition of an adequate number of ancilla qubits to control constantly each V_i (as shown in figure 1), allows us to produce a circuit equivalent of the matrix V in the above equation. Assuming that the first row of V_i is (or includes) i-th row of U, then we must use ($X = N$) as V_i blocks, as shown in equation

$$V|\bar{\psi}\rangle = \begin{pmatrix} V_1 & & \\ & V_2 & \\ & & V_x \end{pmatrix}|\bar{\psi}\rangle = \begin{pmatrix} k\beta_1 \\ \vdots \\ k\beta_2 \\ \vdots \\ k\beta_N \\ \vdots \end{pmatrix} \tag{2}$$

The quantum operations for constructing the matrix V and the operations for modification of the input data $|0\rangle\otimes|\psi\rangle$ form a circuit that simulates the given operator. This means that the steps for formation of the rows of U in V, and also for transformation of $|0\rangle|\psi\rangle$ to $|\bar{\psi}\rangle$ generates basic circuit model for simulating U. One way for the formulation of these steps and to build the matrices V_i and the input data $|\bar{\psi}\rangle$ is as follows: First, the system is expanded by the addition of ancilla qubits. These ancilla qubits constantly control different block quantum operations V_i on the main n qubits (in this report are used n or $(n+1)$ number of ancilla qubits). After forming all elements of U, which we call step of *formation*, the same elements of the rows of U are brought to the first row of each Vi, which we call step of *combination*. The input data are modified ($|0\rangle|\psi\rangle \rightarrow |\bar{\psi}\rangle$) by a small circuit such that $V|\bar{\psi}\rangle$ produces output data which include normalized N states, expected from the operation $U|\psi\rangle$. We call this step *Input modification*. The results from the measurement for these N states exactly simulate $U|\psi\rangle$. The circuit model that will be found with these steps can be mapped out as a block circuit diagram (as shown in fig. 2). This approach ensures a new way for finding the models of the circuits. From here we will describe two different formalized circuit schemes, based on the block circuit from fig. 2.

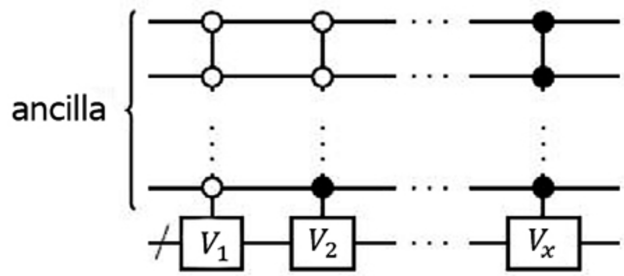

Fig. 1: The number of the qubits on the ancilla determines the number of V_i and hence the amount of V in equation (2).

III. GENERATION OF FORMALIZED CIRCUITS

A. The first model of the circuit

In this model, first we create all elements of *U* in the diagonal positions of *V* through the use of one rotational gate for each element of *U*, step of *formation*. In the step of *combination* the elements of each *i* th row of *U* are collected in the first row of each V_i.

***Step of formation*:** In this step the *U* elements are positioned on the diagonal of a new higher-dimensional matrix V_f . This is a block diagonal matrix with 2 × 2 blocks along the diagonal. For each element of *U* is used one rotational gate. The angular value for the gate is defined for formation of an element of *U* as a value of the cosine. Controlling such gates in a constant binary coded manner produces a matrix which has all elements of *U* on its diagonal:

$$V_f = \begin{pmatrix} R_1 & & \\ & \ddots & \\ & & R_{N^2} \end{pmatrix}_{2N^2 \times 2N^2}, R_j = \begin{pmatrix} c_j & s_j \\ -s_j & c_j \end{pmatrix} \qquad (3),$$

where c_j = cos(arccos(u_j)), generating the *j* element of U. We use *(n +1) number* of all ancilla qubits for constant control of each R_j , $1 \leq j \leq N^2$.

Step of combination: To bring the same elements of *U* of the row to the first rows of V_i, we need a quantum operation V_c, that will produce a matrix $V = V_c V_f$, represented as: (4), where *K* must have a form, looking something like the following matrix: (5). For a system with *(n+1)* qubits, the single gate of Hadamard on the first *n* qubit generates the above matrix with $k = \pm 1/\sqrt{2N}$. From here V_c is a matrix form of this operation in the system with *(2n + 1)* qubits, where we apply the gates of Hadamard to *(n + 1)st*, *n*-th, ..., 3-rd and 2-nd qubits from the bottom of the circuit.

Step input modification: In the final matrix in equation 4, since the respective state for the rows, possessing the elements of *U* with a normalization constant *k*, will be defined as *N* selected states simulating *U*, we must modify the input data in such manner that the elements, represented as "." between ku_{ij} and $ku_{i(j+1)}$ to be ignored. This means that the initial input data must be transformed into $|\bar{\psi}\rangle$ through a preliminary operation to the final matrix *V* so that the relevant elements in the input data to "*" elements should be set to zero: (6), where κ is a normalization constant. It is plain to see that this

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modification can be achieved simply by gates of Hadamard on the first n qubits and the subsequent operations of exchange between the $(n+1)st$ and the remaining n qubits.

The equivalent circuit, stimulating each U, is represented on fig. 3 for n qubit system by the addition of $n + 1$ ancilla qubits and the replacement of the block circuits on fig. 2 with the explicit circuits, indicated above.

At the end of this circuit that may be decomposed in one and two-qubit gates through the use of a technique of decomposition, used in Sec. IV, the following set of N states accurately simulates the given unitary U after normalization: (7), where the dashes are used for separation of the main and ancilla qubits.

In Annex A we give an example for explicit matrices used for each step of the algorithm.

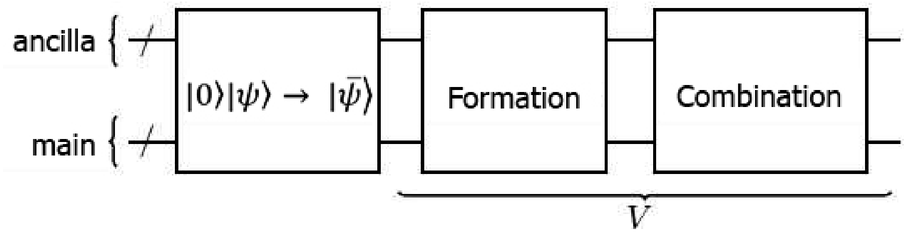

Fig. 2: Diagram of a block circuit for the simulation of U through modification of the input data $|0\rangle|\psi$ to $|\bar{\psi}\rangle$ and construction of V in two steps: the formation of the U elements into V and bringing the same elements in the row in U to the first rows of V_I in V , combination. The necessary gates for the formation of V and also to transform $|0\rangle|\psi$ to $|\bar{\psi}\rangle$ will generate the circuit.

B. The second circuit model

In the first circuit model the elements of U are initially formed on the diagonal of V through the use of constantly controlled rotational gates. Here we take a group of elements from a row of U and we create them as leading rows of small block matrices, as we preserve the relationships between the elements. Using a rotational gate for each two of these small blocks, we create larger block matrices, which will have more U elements in their first rows. This combination of steps is repeated, until the final V_i with leading rows, having rows of U as in equation (2) are constructed. Since the final blocks V_i are $N \times N$, the matrix V is $N_2 \times N_2$; therefore the n are necessary for the ancilla. The step of input modification follows the same idea as the described one for the first model.

Fig. 3: The first model of a circuit for a given general matrix: the initial Hadamards and SWAPs are for modification of the input data, and the last Hadamards transfer the elements to the first rows of V_i (step of combination). The quantum gates in the middle which are constantly controlled form all elements of U on the diagonal of V (step of formation).

Step of formation: As we said above, instead of formation of the matrix elements in the diagonal positions through the use of rotational gates for each element of U, is created a group of elements in the first row of each block with the same rotation as those elements in the original matrix. For example, if the initial blocks are of size 2 by 2, the first row realizes two elements u_{ij} and u_{ik} of U. Thus, the relationship between the first and second element of 2 by 2 block matrix is the same as u_{ij}/u_{ik} (since the block is 2 by 2, the block matrix elements are the values of cosine and sine of angle θ_x , which provides the equality $\cos(\theta_x) / \sin(\theta_x) = u_{ij}/u_{ik}$). In our models of circuits we will assume that $k = j+1$, and thus the first elements of the row of each block realize the ratio of the first elements in the same sequence as the original matrix. Therefore, if the first blocks are of a size $d \times d$, the total number of initial blocks will be N/d, since each block realizes d number of elements. The next matrix represents the step of formation for 2 by 2 initial blocks: (8), where k^i_j are normalization constants and u^i_j are elements of U . The V_i block operations of fig. 4 produce a matrix V_f with 4 by 4 block matrices on its diagonal.

Step of combination: After the formation with ratios, the blocks are combined through the use of one rotational gate for each pair of two blocks, in order to form new larger blocks with new normalization constant that preserve the initial relationships of the elements. Each of these new blocks has twice more elements than the previous blocks. As an example, we will combine 4 by 4 matrices, localized on the diagonal of the matrix V_8 : (9), where k_1 and k_2 are normalization factors. The following matrix V_{c8} can be used as a combined matrix for generation of 8 by 8 larger block from the above pair of two 4 by 4 blocks: (10), where $c_x = cos(\theta_x)$, $s_x = sin(\theta_x)$, and θ_x is an angle to achieve the required ratio. The matrix multiplication $V_{c8}V_8$ produces a matrix with a leading row $[k_x u_1 \ldots k_x u_8]$, where $k_x = sin_x \times k_2$ and $k_x = cos_x \times k_1$.

It's easy to see that the matrix V_{c8} can be written as $R(2\theta_x) \otimes I \otimes I$. From here each main combined matrix can be written as $R \otimes I^D$, where D is the size of the blocks that will be combined through the use of V_c; and R is the main single gubit rotational gate. This means that, for blocks, operating on c qubits, if we apply a rotational gate to the $(c+1)$st qubit, this would be equivalent in matrix form to the operation $V_c V_{2^{c+1}}$. From here the putting of single rotational gates on $(c + 1)st$, $(c + 2)nd$, ..., n-th qubits generates a N by N matrix. In addition, through continuous control of each V_c operation (or equivalent single rotational gates Rs) through the above qubits in the circuit (see the constant controlled rotational gates, localized after V_i block operations on fig. 4) we can generate N as separate blocks and the next final matrix: (11). Since the obtained rows in each block are single vectors and have the same ratio as the elements of the rows of U, they are equal to the relevant rows of U. (The final normalization constant is equal to 1.)

For the general case, if the initial blocks operate on the last c qubit, we must use our $N/2^c$ constantly controlled rotational gates on each main qubit (except for the last c qubit), in order to combine recursively small blocks. In the end we have $N \times N$ blocks, whose leading rows are the U rows, as shown in equation (11).

Input modification $(|0\rangle|\psi\rangle \rightarrow |\bar{\psi}\rangle)$: The modification of the input data $[\alpha_1\ \alpha_2 \ldots \alpha N\ 0 \ldots 0]^T$ as $[\kappa\alpha_1 \ldots \kappa\alpha_N | \kappa\alpha_1 \ldots \kappa\alpha_N | \ldots | \kappa\alpha_1 \ldots \kappa\alpha_N]^T$ with normalization constant κ allows us to stimulate U through the use of V in equation (11) on the selected N states: (12). These input data with $\kappa = 1/\sqrt{2N}$ can be produced by the application of gates of Hadamard to all ancilla qubits at the start of the circuit.

That is why the main model of circuit, shown in fig. 4, is able to simulate any real unitary matrix. For more explicit matrix forms and illustrated details, please refer to Annex A and Annex A2.

3. CONCLUSION

The circuits models listed here are independent of the type of the operator; therefore, they can be used for designing of standard quantum processors and quantum chips, in which the angles are adjusted by a preprocessor element. They may also help in the designing of the possible quantum logical matrices[7]. In addition, since the models of the circuits are highly dependent on the matrix elements, for the application of specific circuits, aimed to fulfill specific types of systems, each level of sparseness in the system can lower the number of the gates significantly in the main model; hence it is possible to be created more effective quantum chips for certain uses. For example, if half of each row of elements are zero in a given matrix, considering the first method, the blocks at the end of the step for combination can be made with the size (N/2×N/2). From here the circuit will require less steps for combination (the number of the qubits in the ancilla is limited to one), which lowers both the classical and CNOT complexity and makes any possible production easier.
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